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Spontaneous decay of an excited atom placed near a rectangular plate

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Abstract. Using the Born expansion of the Green tensor, we consider the spontaneous decay rate of an excited atom placed in the vicinity of a rectangular plate. We discuss the limitations of the commonly used simplifying assumption that the plate extends to infinity in the lateral directions and examine the effects of the atomic dipole moment orientation, atomic position, and plate boundary and thickness on the atomic decay rate. In particular, it is shown that due to the plate finite size, the spontaneous decay may be inhibited even when the atom is situated very close to the surface, and that in the boundary region, the spontaneous decay rate can be strongly modified.

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The ability to control the spontaneous decay process holds the key to powerful applications in micro- and nano-optical devices. Effective control can be achieved by tailoring the environment surrounding the emitters. In theoretical analysis of surrounding environment of different geometries, the most interesting ones being of the resonator type, the boundary conditions are typically taken into account only in directions in which the electromagnetic field is confined or affected the most. For example, in a planar configuration, only the boundary conditions in the normal direction are taken into account while those in the lateral directions are neglected (see, e.g., Ref. [1]). In a cylindrical configuration that extends to infinity, the reverse is true [2]. Under appropriate conditions, these approximations are generally valid. However, as the sizes of devices decrease and fall in the micro- and nano-meter ranges as in the current trend of miniaturization, it is clearly of great importance to keep track of the effects of all boundaries. One way to calculate the spontaneous decay rate in an arbitrary geometry is to directly solve the Maxwell equations using the finite difference time domain method [3]. This method, which relies entirely on numerical computation, is not without weaknesses. It requires that the whole computational domain be gridded, leading to very large computational domains in cases of extended geometries, or in cases where the field values at some distance are required. All curved surfaces must be modeled by a stair-step approximation, which can introduce errors in the results. Additionally, the discretization in time may be a source of errors in the longitudinal field [3].

Here we employ an approach that, in a sense, combines analytical and numerical calculations, thereby significantly reducing the numerical computation workload. This approach relies on first writing the atomic decay rate in terms of the Green tensor characterizing the surrounding media [4,5]. Although this formula holds for arbitrary boundary conditions, exact analytical evaluation of the Green tensors for realistic, finite-size systems can be very cumbersome or even prohibitive. Following [6,7], where the atom-body van der Waals force and the local-field correction, respectively, have been considered, we circumvent the task of an exact calculation of the Green tensor by writing it in terms of a Born series and restrict ourselves to leading-order terms. The boundary conditions enter the theory only via the integral limits. This approach is universal in the sense that it works for an arbitrary geometry of the surrounding media, and can be used to evaluate any characteristics of the matter-electromagnetic field interaction expressible in terms of the Green tensor. In this paper, we are concerned mostly with the spontaneous decay rate of an excited atom placed near a rectangular plate. Our aim is twofold: first, we compare our results with those for an infinitely extended plate in order to establish in a quantitative way the conditions under which the approximation of an infinitely extended plate is valid; second, we examine the effects brought about by the presence of the boundaries in the lateral directions.

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The (classical) Green tensor of an arbitrary dispersing and absorbing body satisfies the equation

$$\hat{H}\boldsymbol{G}(\mathbf{r},\mathbf{r}',\omega) = \delta(\mathbf{r}-\mathbf{r}')\boldsymbol{I}, \tag{1}$$

$$\hat{H}(\mathbf{r}) \equiv \boldsymbol{\nabla}\times\frac{1}{\mu(\mathbf{r},\omega)}\boldsymbol{\nabla}\times - \frac{\omega^2}{c^2}\,\varepsilon(\mathbf{r},\omega), \tag{2}$$

($\boldsymbol{I}$-unit tensor) together with the boundary condition at infinity, where $\varepsilon(\mathbf{r},\omega)$ $[\mu(\mathbf{r},\omega)]$ is the frequency- and space-dependent complex permittivity (permeability) which obeys the Kramers-Kronig relations.

Decomposing the permittivity and permeability as

$$\varepsilon(\mathbf{r},\omega) = \bar{\varepsilon}(\mathbf{r},\omega) + \chi_\varepsilon(\mathbf{r},\omega), \quad \mu(\mathbf{r},\omega) = \bar{\mu}(\mathbf{r},\omega) + \chi_\mu(\mathbf{r},\omega), \tag{3}$$

and assuming that the solution $\bar{\boldsymbol{G}}(\mathbf{r},\mathbf{r}',\omega)$ to the equation $\hat{\bar{H}}(\mathbf{r})\bar{\boldsymbol{G}}(\mathbf{r},\mathbf{r}',\omega) = \delta(\mathbf{r}-\mathbf{r}')\boldsymbol{I}$, where $\hat{\bar{H}}$ is defined as in equation (2) with $\bar{\varepsilon}$ instead of ε and $\bar{\mu}$ instead of μ, is known, the Green tensor $\boldsymbol{G}$ can be written as

$$\boldsymbol{G}(\mathbf{r},\mathbf{r}',\omega) = \bar{\boldsymbol{G}}(\mathbf{r},\mathbf{r}',\omega) + \boldsymbol{G}'(\mathbf{r},\mathbf{r}',\omega). \tag{4}$$

Substituting equation (4) into equation (1) and using the identity $(\bar{\mu}+\chi_\mu)^{-1} = \bar{\mu}^{-1}\sum_{l=0}^{\infty}(\chi_\mu/\bar{\mu})^l$, it can be found that

$$\hat{H}(\mathbf{r})\boldsymbol{G}'(\mathbf{r},\mathbf{r}',\omega) = \hat{H}_\chi(\mathbf{r})\bar{\boldsymbol{G}}(\mathbf{r},\mathbf{r}',\omega) \equiv \tilde{\boldsymbol{G}}(\mathbf{r},\mathbf{r}',\omega) \tag{5}$$

$$\hat{H}_\chi(\mathbf{r}) \equiv -\boldsymbol{\nabla}\times\frac{1}{\bar{\mu}(\mathbf{r},\omega)}\sum_{l=1}^{\infty}\frac{\chi_\mu^l(\mathbf{r},\omega)}{\bar{\mu}^l(\mathbf{r},\omega)}\boldsymbol{\nabla}\times + \frac{\omega^2}{c^2}\chi_\varepsilon(\mathbf{r},\omega), \tag{6}$$

i.e., $\boldsymbol{G}'$ satisfies the same differential equation as the one governing the electric field, with the current being equal to $\tilde{\boldsymbol{G}}$. Hence it can be written as a convolution of this current with the kernel $\boldsymbol{G}$: $\mathbf{G}'(\mathbf{r},\mathbf{r}',\omega) = \int d^3s\boldsymbol{G}(\mathbf{r},\mathbf{s},\omega)\tilde{\boldsymbol{G}}(\mathbf{s},\mathbf{r}',\omega)$. Substitution of $\boldsymbol{G}'$ in this form in equation (4) and iteration lead to the desired Born series

$$\boldsymbol{G}(\mathbf{r},\mathbf{r}',\omega) = \bar{\boldsymbol{G}}(\mathbf{r},\mathbf{r}',\omega) + \sum_{k=1}^{\infty}\boldsymbol{G}_k(\mathbf{r},\mathbf{r}',\omega), \tag{7}$$

$$\boldsymbol{G}_k(\mathbf{r},\mathbf{r}',\omega) = \left(\prod_{j=1}^{k}\int \mathrm{d}^3 s_j\right) \times \bar{\boldsymbol{G}}(\mathbf{r},\mathbf{s}_1,\omega)\tilde{\boldsymbol{G}}(\mathbf{s}_1,\mathbf{s}_2,\omega)\cdots\tilde{\boldsymbol{G}}(\mathbf{s}_k,\mathbf{r}',\omega)\,. \tag{8}$$

This formal expansion for the Green tensor is valid for an arbitrary geometry, permittivity, and permeability of the macroscopic bodies. Obviously, one of the situations in which the Born series is particularly useful is when χ_ε is a perturbation to $\bar{\varepsilon}$ and χ_μ is a perturbation to $\bar{\mu}$, thereby one makes only a small error cutting off higher-order terms. For such weakly dielectric and magnetic bodies, it is natural to choose

$$\bar{\varepsilon}(\mathbf{r},\omega) = \bar{\mu}(\mathbf{r},\omega) = 1 \tag{9}$$

with $\chi_\lambda(\mathbf{r},\omega) = \chi_{\lambda\mathrm{R}}(\mathbf{r},\omega) + i\chi_{\lambda\mathrm{I}}(\mathbf{r},\omega)$, $|\chi_\lambda(\mathbf{r},\omega)| \ll 1$ $(\lambda = \varepsilon,\ \mu)$ [cf. Eqs. (3)]. This implies we restrict ourselves to frequencies far from a medium resonance.

The Green tensor $\bar{\boldsymbol{G}}$ corresponding to $\bar{\varepsilon}(\mathbf{r},\omega) = \bar{\mu}(\mathbf{r},\omega) = 1$ is the vacuum Green tensor

$$\bar{\boldsymbol{G}}(\mathbf{r},\mathbf{r}',\omega) = -\frac{\delta(\mathbf{u})}{3k^2}\boldsymbol{I} + \frac{k}{4\pi}(a\boldsymbol{I} - b\hat{\mathbf{u}}\otimes\hat{\mathbf{u}})e^{iq}, \tag{10}$$

$$a \equiv a(q) = \frac{1}{q} + \frac{i}{q^2} - \frac{1}{q^3}, \quad b \equiv b(q) = \frac{1}{q} + \frac{3i}{q^2} - \frac{3}{q^3} \tag{11}$$

($k = \omega/c$; $\mathbf{u} \equiv \mathbf{r}-\mathbf{r}'$; $\hat{\mathbf{u}} = \mathbf{u}/u$, and $q \equiv ku$).

A description of quantities such as the emission pattern or the interatomic van der Waals forces requires the knowledge of the Green tensor of different positions, while a description of quantities such as the spontaneous decay rate of an excited atom or the atom-body van der Waals forces requires the knowledge of the Green tensor of equal positions. Substituting equation (10) in equation (8) and assuming that the position $\mathbf{r}$ lies outside the region occupied by the macroscopic bodies, we derive for the first-order term in the Born expansion of the equal-position Green tensor

$$\boldsymbol{G}_1(\mathbf{r},\mathbf{r},\omega) = \frac{k^2}{16\pi^2}\int \mathrm{d}^3 s\,\hat{H}_{\chi 1}(\mathbf{s}) \times [a^2\boldsymbol{I} + (b^2-2ab)\hat{\mathbf{u}}\otimes\hat{\mathbf{u}}]e^{2iq} \tag{12}$$

[$\mathbf{u} \equiv \mathbf{r}-\mathbf{s}$; $q = ku$; $a = a(q)$; $b = b(q)$; $\hat{H}_{\chi 1}(\mathbf{r}) \equiv -\boldsymbol{\nabla}\times\chi_\mu(\mathbf{r},\omega)\boldsymbol{\nabla}\times + \frac{\omega^2}{c^2}\chi_\varepsilon(\mathbf{r},\omega)$ – linear part of $\hat{H}_\chi$, Eq. (6)]. Higher-order terms can easily be obtained by repeatedly using equation (10) in equation (8).

Our system consists of an excited two-level atom surrounded by macroscopic media, which can be absorbing and dispersing. In the electric-dipole and rotating-wave approximations, the atomic decay rate reads as [4,5]

$$\Gamma = \frac{2k_\mathrm{A}^2}{\hbar\varepsilon_0}\,\mathbf{d}_\mathrm{A}\mathrm{Im}\,\boldsymbol{G}(\mathbf{r}_\mathrm{A},\mathbf{r}_\mathrm{A},\omega_\mathrm{A})\mathbf{d}_\mathrm{A}, \tag{13}$$

where $\mathbf{d}_\mathrm{A}$ and ω_A are the atomic dipole and shifted transition frequency, $k_\mathrm{A} = \omega_\mathrm{A}/c$, and $\boldsymbol{G}$ is the Green tensor taken at the position of the atom and at the atomic transition frequency.

In accordance with the linear Born expansion, equations (7), (10), and (12) yield, for a purely electric material,

$$\frac{\Gamma^{\parallel(\perp)}}{\Gamma_0} = 1 + \frac{3k_\mathrm{A}^3}{8\pi}\mathrm{Im}\left\{\int \mathrm{d}^3 s\,\chi_\varepsilon(\mathbf{s},\omega_\mathrm{A}) \times \left[a^2 + (b^2-2ab)\frac{1}{u^2}\begin{matrix}(x-x_\mathrm{A})^2\\(z-z_\mathrm{A})^2\end{matrix}\right]e^{2iq}\right\} \tag{14}$$

[$\Gamma_0 = k_\mathrm{A}^3 d_\mathrm{A}^2/(3\pi\hbar\varepsilon_0)$ – free-space decay rate, $\mathbf{s} = (x,y,z)$, $u = |\mathbf{s}-\mathbf{r}_\mathrm{A}|$, $q = k_\mathrm{A}u$, $a = a(q)$, $b = b(q)$] for x-(z-)oriented dipole moments. Equations (14) are our main working equations. Just like the Born expansion of the Green tensor, they hold for an arbitrary geometry of the surrounding environment.

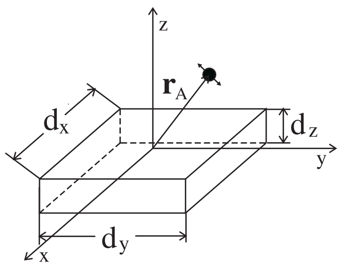

Fig. 1. A dipole emitter in the vicinity of a rectangular plate.

Next let us be specific about the shape of the macroscopic bodies. We consider a rectangular plate of dimensions d_x, d_y, and d_z and choose a Cartesian coordinate system such that its origin is located at the center of one surface of the plate, as sketched in Figure 1. Then $\Gamma^{\parallel}$ represents the spontaneous decay rate of a dipole moment parallel to a plate surface and $\Gamma^{\perp}$ – that of a dipole moment normal to the same surface.

With the integral limits in equation (14) specified, one can use the stationary phase method to obtain (Appendix A)

$$\frac{\Gamma^{\parallel(\perp)}}{\Gamma_0} \simeq 1 + \frac{3k_A^3}{8\pi}\mathrm{Im}\Bigg[\chi_\varepsilon \int_0^{d_z} \mathrm{d}z\, \frac{a^2}{(a-b)^2} e^{2iq_z} \times \int_{-\frac{d_x}{2}}^{\frac{d_x}{2}} \mathrm{d}x e^{ik_A^2(x-x_A)^2/q_z} \int_{-\frac{d_y}{2}}^{\frac{d_y}{2}} \mathrm{d}y\, e^{ik_A^2(y-y_A)^2/q_z}\Bigg] \quad (15)$$

[$a = a(q_z)$, $b = b(q_z)$, $q_z = k_A(z + z_A)$], provided that $-\frac{d_x}{2} \leq x_A \leq \frac{d_x}{2}$ and $-\frac{d_y}{2} \leq y_A \leq \frac{d_y}{2}$. In the particular case of $x_A = y_A = 0$, equations (15) become

$$\frac{\Gamma^{\parallel(\perp)}}{\Gamma_0} \simeq 1 + \frac{3k_A^3}{2\pi}\mathrm{Im}\Bigg[\chi_\varepsilon \int_0^{d_z} \mathrm{d}z\, \frac{a^2}{(a-b)^2} e^{2iq_z} \times \int_0^{\frac{d_x}{2}} \mathrm{d}x e^{ik_A^2 x^2/q_z} \int_0^{\frac{d_y}{2}} \mathrm{d}y\, e^{ik_A^2 y^2/q_z}\Bigg], \quad (16)$$

where the integrals over x and y contain Fresnel integrals [8]. For small lateral sizes such that $d_x, d_y \ll \sqrt{z_A/\lambda_A}\,\lambda_A$, using the series expansions of the Fresnel integrals [8] and keeping only the leading terms, we derive the following expressions

$$\frac{\Gamma^{\parallel(\perp)}}{\Gamma_0} \simeq 1 + \frac{3k_A^3}{8\pi} d_x d_y \mathrm{Im}\left[\chi_\varepsilon \int_0^{d_z} \mathrm{d}z\, \frac{a^2}{(a-b)^2} e^{2iq_z}\right], \quad (17)$$

which indicate a linear dependence of the decay rates on d_x and d_y. Obviously, both $\Gamma^{\parallel}$ and $\Gamma^{\perp}$ approach the free-space value as d_x and/or d_y tend to zero. In what follows, we resort to numerical computation.

In Figure 2 we present the spontaneous decay rate in accordance with the linear Born expansion (14), as a function of the atom-surface distance for two different values of the permittivity. The same quantity but for an atom placed near an infinitely extended planar slab is plotted using the Green tensor given in reference [9]. It can be seen that when the lateral dimensions of the rectangular plate are sufficiently large and the absolute value of the permittivity is sufficiently close to one (the case of $\chi_\varepsilon = 0.1 + i10^{-8}$ in the figure), the two results almost coincide for both dipole moment orientations. As $\chi_{\varepsilon R}$ increases, the agreement worsens but is still quite good at $\chi_{\varepsilon R} = 0.5$. Further computations indicate that in the range of $|\chi_\varepsilon(\omega_A)| \lesssim 0.5$, the spontaneous decay rate can be well approximated by the linear Born expansion, and we shall stay within this range in numerical examples given below. In Figure 2, the atom has been moved along the z-axis with $x_A = y_A = 0$. When the atom is moved along other lines parallel to the z-axis but nearer to the border, the z_A-dependence of the normalized spontaneous decay rates behaves in a similar way as in Figure 2 but with its value being generally closer to one.

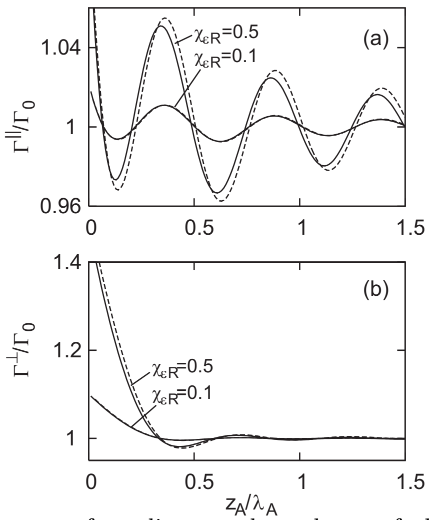

Fig. 2. Atom-surface distance dependence of the normalized spontaneous decay rate of an excited atom positioned at $(0, 0, z_A)$ near an infinite planar plate (solid line) and a rectangular plate ($d_x = d_y = 10\lambda_A$, dashed line) of equal thickness $d_z = 0.2\lambda_A$ and equal $\chi_\varepsilon = \chi_{\varepsilon R} + i10^{-8}$. Case (a) is for a x-oriented dipole moment, while case (b) is for a z-oriented dipole moment.

In Figure 3, we gradually reduce the lateral sizes of the rectangular plate while keeping its thickness constant, and compare the resulting spontaneous decay rates with that for an infinite slab. The permittivity is set equal to $\varepsilon(\omega_A) = 1.1 + i10^{-8}$ – a value which is very close to one (cf. Fig. 2). For lengths of the lateral sides comparable to the atomic transition wavelength, the infinite slab approximation starts to differ noticeably from the linear Born expansion (see the figure, case of $d_x = d_y = 3\lambda_A$), which in this situation is regarded as a good and nondegrading approximation. As the lateral sizes of the plate decrease further and become smaller than λ_A, the infinite-slab approximation fails completely (see cases of $d_x = d_y = 0.4\lambda_A$ and $0.2\lambda_A$ in the figure). In other words, while a rectangular plate with lateral sizes much larger than the atomic transition wavelength can be more or less treated as an infinite slab, care should be taken when the sizes are reduced

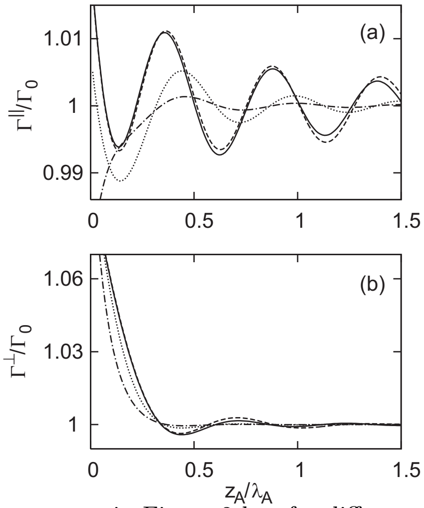

Fig. 3. The same as in Figure 2 but for different sizes of the rectangular plate: $d_x = d_y = 3\lambda_A$ (dashed line), $0.4\lambda_A$ (dotted line), and $0.2\lambda_A$ (dash-dotted line). In the last case, the plate is actually a cube. The curves for an infinite planar plate are shown by solid line and $\chi_\varepsilon = 0.1 + i10^{-8}$.

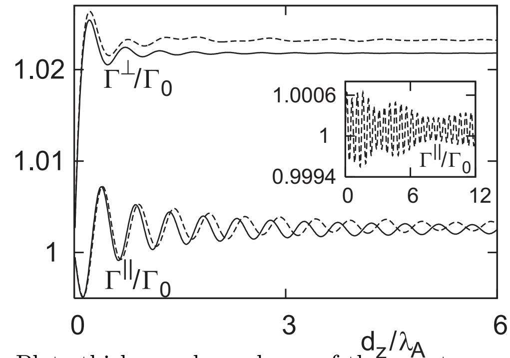

Fig. 4. Plate-thickness dependence of the spontaneous decay rate of an excited atom located near an infinitely extended planar plate (solid line) and a rectangular plate ($d_x = d_y = 10\lambda_A$, dashed line) of equal $\chi_\varepsilon = 0.1 + i10^{-8}$. The atomic position is fixed at $(0, 0, 0.2\lambda_A)$ in the main figure, and $(0, 0, 5\lambda_A)$ in the inset.

to about or below a wavelength. This happens for both normal and parallel to the surface dipole moment orientations. When the rectangular plate can be roughly regarded as an infinite slab, the infinite-slab curve and the linear Born expansion curve agree better when the atom is placed closer to the surface (compare dashed and solid curves in Fig. 3).

The dash-dotted curve in Figure 3a for an atomic dipole moment oriented parallel to the xy-surface demonstrates an unusual phenomenon which is completely absent in infinitely extended systems. Namely, the spontaneous decay is suppressed even in the close-to-the surface limit $z_A/\lambda_A \ll 1$. For dipole moments oriented normal to the xy-surface, only a reduction of the spontaneous-decay enhancement is observed [Fig. 3b]. The strong enhancement of the spontaneous decay near the surface of infinite slabs is often explained as being due to the coupling of the atom to the evanescent waves whose amplitudes are large around the surface and exponentially decrease away from it [1]. These evanescent waves propagate along the surface. When the slab has a finite size, we speculate that the evanescent waves are reflected, at least partially, when encountering a boundary, giving rise to interference effects that lead to the suppression of the spontaneous decay mentioned above. When the lateral size of the slab is reduced even further, numerical computations confirm the fact that the spontaneous decay rates tend to that in free space [cf. Eqs. (15)–(17)].

Besides the dependence on the lateral sizes, whether a rectangular plate can be treated as an infinitely extended one clearly depends on its thickness as well. When the material absorption is negligible, it is intuitively obvious that even for plate lateral sizes much larger than the atomic transition wavelength, the plate cannot be treated as extending to infinity if its thickness is comparable with the lateral sizes. In Figure 4, we compare the d_z-dependence of the spontaneous decay rate for a rectangular plate with that for an infinite slab. The agreement between the two curves, being very good for sufficiently thin plates, gradually worsens with an increasing plate thickness. The disagreement is already noticeable at $d_z \sim \lambda_A$ – a value which is still much smaller than the lateral sizes $d_x = d_y = 10\lambda_A$, and it sets in earlier for $\Gamma^{\perp}$ than for $\Gamma^{\|}$. The two calculations predict quite different large-thickness limits. This means that in the case of thick plates, one must take into account the boundary conditions in the lateral directions in order to obtain reliable results.

Account of the boundary conditions in the lateral directions also gives rise to some curious beating in the d_z-dependence of the spontaneous decay rate, which is especially visible when the atom has a dipole moment oriented parallel to the surface and is situated somewhat away from the surface (see Fig. 4, inset). Let's have a closer look at, say, the decay rate for a dipole oriented parallel to the surface. In the limit of an infinite plate $d_x,\ d_y \to \infty$, $\Gamma^{\|}$ in Eqs. (16) becomes

$$\frac{\Gamma^{\|}}{\Gamma_0} \simeq 1 + \frac{3k_A}{16}\mathrm{Im}\left[\chi_\varepsilon \int_0^{d_z} \mathrm{d}z a^2(q_z) q_z e^{2iq_z}(1+i)^2\right]. \quad (18)$$

As a function of z/λ_A, the integrand in equation (18) has a period of $\frac{1}{2}$. Since this period is z-independent, the resulting integral must exhibit oscillations with the same period, as confirmed by Figure 4, solid curves. These oscillations survive for plates of finite lateral extensions (Fig. 4, dashed curves). The beating clearly arises from the finite values of d_x and d_y. Note that as a function of z, the inner integrands in equation (16) have a 'period' that is z-dependent and that increases with increasing z.

Next we turn to elucidating the influence of the boundaries in the x- and y-directions on the spontaneous decay rates. As can be seen from Figure 5, where an edge is present at $x_A = 5\lambda_A$, the decay rates exhibit oscillations near the boundary with a particularly strong magnitude right on either side of it, and damping tails. The oscillations are more pronounced for a dipole moment oriented parallel to the (xy)-plane than for a z-oriented

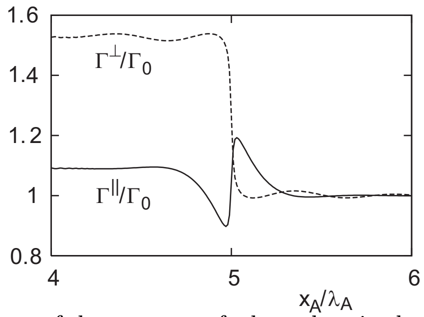

Fig. 5. Effects of the presence of a boundary in the x-direction on the spontaneous decay rate of an excited atom located near a rectangular plate of permittivity $\varepsilon(\omega_\mathrm{A}) = 1.5 + i10^{-8}$ and dimensions $d_z = 0.2\lambda_\mathrm{A}$ and $d_x = d_y = 10\lambda_\mathrm{A}$. The atom is located at $(x_\mathrm{A}, 0, 0.01\lambda_\mathrm{A})$.

dipole moment. One can notice that when the projection of the atomic position on the xy-plane lies outside and sufficiently far from the boundaries, the spontaneous decay rate approaches one in free space, as it should.

In summary, using the Born expansion of the Green tensor, we have considered the decay rate of an atom located near a plate of rectangular shape. We have shown that a rectangular plate can be treated as extending to infinity only when its lateral sizes are much larger than the atomic transition wavelength, its thickness sufficiently small, and the atom is located close enough to the plate surface. The smallness of the lateral size of the plate may lead to an inhibition of the spontaneous decay even when the atom is located very close to the surface – in contrast to the strong enhancement observable in infinitely extended systems. We have also shown that a boundary in the lateral directions can give rise to significant modifications of the decay rate in either side of it. The first-order Born expansion remains quite reliable even at a value of permittivity as high as 1.5. Inclusion of higher-order terms would allow one to investigate more dense media.

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Appendix A: Derivation of equations (15)

For an atom located near a rectangular plate, the decay rates (14) read as

$$\frac{\Gamma^{\|(\perp)}}{\Gamma_0} = 1 + \frac{3k_\mathrm{A}^3}{8\pi}\mathrm{Im}\Biggl\{\chi_\varepsilon \int_{-d_z}^{0} \mathrm{d}z \int_{-\frac{d_x}{2}}^{\frac{d_x}{2}} \mathrm{d}x \int_{-\frac{d_y}{2}}^{\frac{d_y}{2}} \mathrm{d}y \times \left[a^2 + (b^2 - 2ab)\frac{1}{u^2}\frac{(x - x_\mathrm{A})^2}{(z - z_\mathrm{A})^2}\right] e^{2iq}\Biggr\}. \quad \text{(A.1)}$$

The integrand in equation (A.1) contains the exponential function $\exp(2iq)$, which oscillates with the phase $2q$, where

$$q = k_\mathrm{A}\sqrt{(x - x_\mathrm{A})^2 + (y - y_\mathrm{A})^2 + (z - z_\mathrm{A})^2}. \quad \text{(A.2)}$$

According to the idea of the stationary phase method, which relies on the cancellation of sinusoids with rapidly-varying phase, the main contribution to the integrals is from the region where the phase is stationary

$$\frac{\partial q}{\partial x} = 0, \quad \text{(A.3)}$$

$$\frac{\partial q}{\partial y} = 0, \quad \text{(A.4)}$$

$$\frac{\partial q}{\partial z} = 0, \quad \text{(A.5)}$$

which, for q determined as in equation (A.2), imply

$$x - x_\mathrm{A} = 0, \quad \text{(A.6)}$$

$$y - y_\mathrm{A} = 0, \quad \text{(A.7)}$$

$$z - z_\mathrm{A} = 0. \quad \text{(A.8)}$$

The condition (A.8) cannot be fulfilled because the atom is located outside the plate: $z < 0$ while $z_\mathrm{A} > 0$. For the conditions (A.6) and (A.7) to be satisfied, it is required that $-\frac{d_x}{2} \le x_\mathrm{A} \le \frac{d_x}{2}$ and $-\frac{d_y}{2} \le y_\mathrm{A} \le \frac{d_y}{2}$. Since the main contribution to the integrals is for values of x and y about x_A and y_A, respectively, one can write

$$q \simeq k_\mathrm{A}|z - z_\mathrm{A}|\left[1 + \frac{(x - x_\mathrm{A})^2}{2(z - z_\mathrm{A})^2} + \frac{(y - y_\mathrm{A})^2}{2(z - z_\mathrm{A})^2}\right] = q_z + \frac{k_\mathrm{A}^2(x - x_\mathrm{A})^2}{2q_z} + \frac{k_\mathrm{A}^2(y - y_\mathrm{A})^2}{2q_z}, \quad \text{(A.9)}$$

where $q_z = k_\mathrm{A}|z - z_\mathrm{A}|$.

Substituting $x = x_\mathrm{A}$ and $y = y_\mathrm{A}$ in the nonoscillating factor in front of the exponential in the integral, using q as in equation (A.9) in the exponential, and changing the variable z to $-z$, we arrive at equations (15).

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